PBS Exam Protocol SS17

Mittwoch, 14. Februar 2018 20:56

15min Oral exam. Present: M. Bächer, V. da Costa de Azevedo
Protocol written from my recollection at the evening of the exam day. That's more than 11 hours after my exam, so I might have forgotten some questions. I have omitted the parts where they affirmatively nodded or signaled to continue.
Achieved Grade: 5

Pt.1 by Prof. Bächer (Please assume any content is not factually correct.)
What is the Deformation Gradient for a Translation?
I didn't really look at those slides, I thought that topic was irrelevant. But I assume the Deformation Gradient is about how the deformation changes… So it's probably zero because we are looking at rigid bodies.
Yes....but we used matrixes
Then that would be the identity matrix.
Please write that down
* I write down the 2 × 2 identity matrix after verifying that he doesn't ask for homogenuous coordinate representation (which wouldn't be part of this lecture anyways, I think) *

How about a 2D Rotation?
Well, I know the 3D Rotation-matrix by heart, so I could just…
No, I want it in 2D
* I hesitate. He points out that there's an equation on the printed-out slide that he's showing me*

$$x = \cos\theta\ X\ - \sin\theta\ Y$$
$$y = \sin\theta\ X\ - \cos\theta\ Y$$

We want to get x and y from X and Y.
Oh, okay. * I write that down as a matrix. I think I forgot the minus signs, but he didn't notice either. *
What is the deformation gradient now?
* It took me a while to figure out that he wanted me to point at the matrix I just wrote down. *
Is this irrotational?
dunno….
No. Is that a problem?
I guess not….
Yes it is a problem. Because it shouldn't depend on rotation. (I'm still not sure what this 'it' was but ok)

*Let us talk about this slide now (* points out another printed slide which is basically empty except for an epsilon and some other sign which are waiting for an equation *). We looked at two kinds of strain in the lecture…*?
Either you're talking about stress and strain, or about Green and Cauchy strain.
The latter. Formulas plz.
I don't know those. Green strain is exact and Cauchy strain is linear. This causes some artifacts when we only use the Cauchy strain.
Yes. But formulas plz. What do they look like?
I don't know them. Cauchy strain probably lacks a few squared terms.
How about irrotationality?
… I guess it changes when you rotate it
… Please write down for me. "F".
So, is that the force?
No, it's about the deformation gradient. You can write it as F=RU. Do you know this decomposition?
Well, I know of the LU decomposition… maybe R is a rotation though.
It's kinda that. But R is a Rotation here and U is the strain and stress. You can write it as $F^T F$. Please do so.
$F^T F = (RU)^T (RU) = U^T R^T R U$ and R is a rotation matrix so it is orthogonal and the whole middle

part becomes zero. So we're left with $U^T U$. I assume those are not orthogonal matrixes.
Right, they're not. So what does this mean?
Well.. we could match $F = U$. But that doesn't have to hold.
Yeah but you made the R disappear
Oh. Yeah that means it's irrotational.

Pt.2 By V. da Costa de Azevedo
Would you rather talk about (I forgot what. Probably Rigid Bodies) or Fluids?
Fluids
We have covered the nabla operator. What operations can we define with it?
The cross product is rotation. The dot product is divergence. Or we could just apply the nabla operator directly.
What is that called?
Applying the nabla operator… or the gradient. (That's the word he wanted to hear)
*And what happens if you do this? * Writes down $\nabla \cdot \nabla F$ **
* I'm momentarily confused because I don't know what that is called. *
I don't know right now (Hoping to proceed faster to the interesting stuff that I actually studied.)
The laplacian (Should've known that)
*We had two equations in class. one for mass co… *hesitates**
One was mass conservation and the other was momentum conservation. We could derive the momentum conservation from the material derivative by… (he interrupts me.)
Nono, we don't go that deep. Now… We had some equation for fluids…
The Navier-Stokes equations?
Yes. Do you know the Helmholtz decomposition?
The helmholtz theorem states that for any vector field u, there is a way to decompose it into $\vec{u} = \nabla\varphi + \nabla \times \psi + h$. We have always treated this as harmonic, i.e. $h = 0$. $\nabla\varphi$ is the irrotational part and $\nabla \times \psi$ is the incompressible part. That means $\nabla(\nabla \times \psi)$ equals 0.
Do you know the mass conservation? It was the less complicated one.
Yes. $\nabla u = 0$
Could you apply the nabla operator to the helmholtz equation?
This gives us $\nabla u = \nabla^2\varphi + 0 + 0$. We could use this to update the… I tend to switch up incompressible and irrotational, but THIS part of the navier stokes equation, which is the pressure: $-\frac{1}{\rho}\nabla p$.
Wait wait, so what is the pressure in the helmholtz equation?
It's φ.
So…? What is it up here?
Uhm, we could match it with the irrotational part. But matching is a bit of magic because Helmholtz decomposition is not guaranteed to be unique. It could be just..
Uhm. Yes. But we match here anyways.
So you have that equation $\nabla u = \nabla^2\varphi$. How do you solve this for u with the pressure?
We would usually do that the other way around. First, advect the velocities, then use this equation with the pressure for φ to make the whole system match again - because for that equation we assumed incompressibility. So we restore incompressibility.
* I notice that he is still waiting for something. Prof. Bächer seems to be eyeing me curiously because I suddenly know shit. *
We then project the velocities with $\nabla u = -\nabla p$ because the velocities should point more towards where the pressure becomes lower.
Could you explain semi-lagrangian advection to me?
We start with a staggered grid, that means we have the velocities on the centers of the grid cell borders and the pressures in the middle. This avoids the checkerboard problem which we would have otherwise because..
Yes, yes.
So we imagine a particle at every place where we want to know the velocity in the new time and try to find out where it came from by moving in the direction of the negative velocity. We could either use euler or second-order runge-kutta method for that. We then take the velocity and use that as the new velocity because we assume that a particle keeps its velocity. We do the same for density..

and
Do we do that for pressure as well?
No. Only for the density and velocity. We later compute the pressure with... * I'm unsure, but he deems this to be sufficient. Possibly because the time was up. We had already started at least 10min later than expected. *
Alright, time's up. Please tell the next one that we need half a minute.